

# Predictability of extreme events in social media

José M. Miotto<sup>1,\*</sup>, Eduardo G. Altmann<sup>1</sup>

**1 Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany**

\* E-mail: jmiotto@pks.mpg.de

## Abstract

It is part of our daily social-media experience that seemingly ordinary items (videos, news, publications, etc.) unexpectedly gain an enormous amount of attention. Here we investigate how unexpected these extreme events are. We propose a method that, given some information on the items, quantifies the predictability of events, i.e., the potential of identifying in advance the most successful items. Applying this method to different data, ranging from views in YouTube videos to posts in Usenet discussion groups, we invariantly find that the predictability increases for the most extreme events. This indicates that, despite the inherently stochastic collective dynamics of users, efficient prediction is possible for the most successful items.

## Introduction

When items produced in social media are abundant, the public attention is the scarce factor for which they compete [1–3]. Success in such *economy of attention* is very uneven: the distribution of attention across different items typically shows heavy tails which resemble Pareto’s distribution of income [4] and, more generally, are an outcome of complex collective dynamics [5–12] and non-trivial maximizations of entropic functions [13, 14]. Increasing availability of large databases confirm the universality of these observations and renew the interest on understanding the dynamics of attention, see Tab. 1.

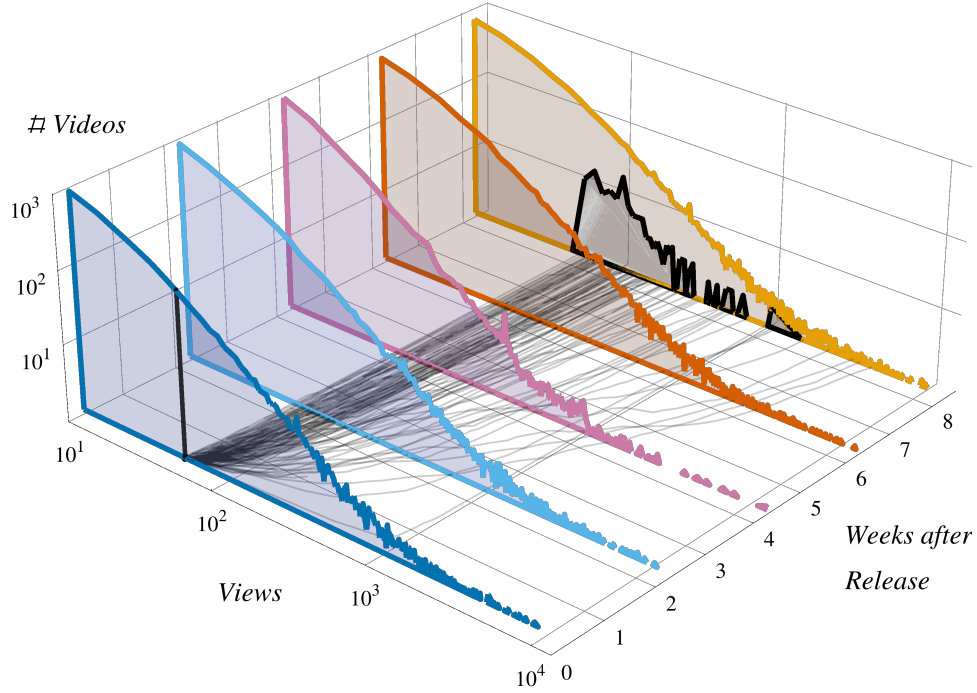
**Table 1. Examples in which fat-tailed distributions of popularity across items have been reported.**

System	Item	Attention measure	Refs.
Online Videos	video	views, likes	[15]
Discussion Groups	threads	posts, answers	[16]
Publications	papers	citations, views	[6, 8, 17, 18]
Twitter	tweet	retweets	[9]
WWW	webpage	views	[11]
Online Petitions	petition	signers	[19]

Universal features of heavy-tailed distributions do not easily lead to a good forecast of specific items [5], a problem of major fundamental and practical interest [15, 17, 18, 20, 21]. This is illustrated in Fig. 1, which shows that the heavy-tailed distribution appears at very short times but items with the same early success have radically different future evolutions. The path of each item is sensitively dependent on idiosyncratic decisions which may be amplified through collective phenomena.

An important question is how to quantify the extent into which prediction of individual items is possible (i.e., their *predictability*) [22]. Of particular interest –in social and natural systems– is the predictability of extreme events [23–28], the small number of items in the tail of the distribution that gather a substantial portion of the public attention.

Measuring predictability is difficult because it is usually impossible to disentangle how multiple factors affect the quality of predictions. For instance, predictions of the attention that individual items are going to receive rely on (i) information on properties of the item (e.g., metadata or the attention received in the



**Figure 1. Dynamics of views in YouTube.** **Colored histograms:** distributions of views at fixed times after publication (0.3 million videos from our database). **Gray lines at the bottom:** trajectories of 120 videos which had the same early success (50 views 2 days after publication). **Black histogram:** distribution of views of the 120 selected videos 2 months after publication.

first days) and (ii) a prediction strategy that converts the information into predictions. The quality of the predictions reflect the interplay between these two factors and the dynamics of attention in the system. In particular, the choice of the prediction strategy is crucial. Instead, predictability is a property of the system and is by definition independent of the prediction strategy (it is the upper bound for the quality of any prediction based on the same information on the items). A proper measure of the predictability should provide direct access to the properties of the system, enabling a quantification of the importance of different information on the items in terms of their predictive power.

In this paper we introduce a method to quantify the predictability of extreme events and apply it to data from social media. This is done by formulating a simple prediction problem which allows for the computation of the optimal prediction strategy. The problem we consider is to provide a binary (yes/no) prediction whether an item will be an extreme event or not (attention passes a given threshold). Predictability is then quantified as the quality of the optimal strategy. We apply this method to four different systems: views of YouTube videos, comments in threads of Usenet discussion groups, votes to Stack-Overflow questions, and number of views of papers published in the journal PLOS ONE. Our most striking empirical finding is that in all cases the predictability increases for more extreme events (increasing threshold). We show that this observation is a direct consequence of differences in (the tails of) the distributions of attention conditioned by the known property about the items.

The paper is divided as follows: Sec. Motivation motivates the problem of event prediction by showing that it is robust to data with heavy tails. Sec. Methods introduces the method to quantify predictability, which is used in the Sec. Application to Data. A summary of our findings appears in Sec. Conclusions.

## Motivation

### Characterization of Heavy-tails

Different systems in which competition for attention takes place share similar statistical properties. Here we quantify attention of published items in 4 representative systems (see Sec. 1 of the Supporting Information (SI) for details; all the data is available in Ref. [29]):

- views received by 16.2 million videos in YouTube.com between Jan. 2012 and Apr. 2013;
- posts written in 0.8 million threads in 9 different Usenet discussion groups between 1994 and 2008;
- votes to 4.6 million questions published in Stack-Overflow between Jul. 2008 and Mar. 2013.
- views of 72246 papers published in the journal PLOS ONE from Dec. 2006 to Aug. 2013 (see also Ref. [30]).

The tails of the distribution  $P(X)$  of attention  $X$  (views, posts, etc.) received by the items (videos, threads, etc.) at a large time  $t$  after publication is characterized without loss of generality using Extreme Value Theory. It states that for large thresholds  $x_p$  the probability  $P(X|X > x_p)$  follows a Generalized Pareto distribution [31]

$$P(X > x|X > x_p) \sim \left(1 + \frac{x - x_p}{\sigma\alpha}\right)^{-\alpha}. \quad (1)$$

The fits of different partitions of our databases yield  $\alpha \in [0.50, 4.36]$  and are statistically significant already for relatively small  $x_p$ 's ( $p$ -value  $> 0.05$  in 52 out of 59 fits, see SI Sec. 2 and Fig. S1 for details). These results confirm the presence of heavy tails, an observation reported previously in a variety of cases (see Tab. 1). This suggests that our databases are representative of social media more generally (while scientific publications are usually not classified as social media items, from the point of view of their online views, they are subject to the same attention-gathering process).

### Prediction of Extreme Events

Prediction in data with heavy tails is typically not robust. As an example, consider using as a predictor  $\hat{X}$  of the future attention the mean  $\hat{X} = \sum_{x=1}^{\infty} xP(x)$ , which is the optimal predictor, if we measure the quality of prediction with the standard deviation of  $X$ . For heavy-tailed distributions, the mean and standard deviation may not be defined (for  $\alpha < 1$  and  $\alpha < 2$ , respectively), making prediction not robust (i.e., it depends sensitively on the training and target datasets). This illustrates the problems heavy-tails typically appear when value predictions are issued and indicates the need for a different approach to prediction of attention.

We consider the problem of *event prediction* because, as shown below, it is robust against fat-tailed distributions. We say an event  $E$  happens at time  $t$  if the cumulative attention  $X(t)$  received by the considered item until time  $t$  is within a given range of values. We are particularly interested in predicting extreme events  $X(t) > x_*$ , i.e., to determine whether the attention to an item passes a threshold  $x_*$  before time  $t$ . The variable to be predicted for each item is binary:  $E$  or  $\bar{E}$  (not  $E$ ). We consider the problem of issuing binary predictions for each item ( $E$  will occur or not), which is equivalent to a classification problem and different from a probabilistic prediction ( $E$  will occur with a given probability). Heavy tails do not affect the robustness of the method because all items for which  $X(t) > x_*$  count the same (each of them as one event), regardless of their size  $x$ . Indeed, the tails of  $P(X > x_*)$  determine simply how the probability of an event  $P(E)$  depends on the threshold  $x_*$  (we assume  $P(X)$  exists).

## Methods

In this section we introduce a method to quantify predictability based on the binary prediction of extreme events. This is done by arguing that, despite the seeming freedom to choose between different prediction strategies, it is possible to compute a single optimal strategy for this problem. We then show how the quality of prediction can be quantified and argue that the quality of the optimal strategy is a proper quantification of predictability.

Predictions are based on information on items which generally lead to a partition of the items in groups  $g \in \{1, \dots, G\}$  that have the same feature [32]. As a simple example of our general approach, consider the problem of predicting at publication time  $t = 0$  the YouTube videos that at  $t = t_* = 20$  days will have more than  $x_* = 1000$  views (about  $P(E) \approx 6\%$  of all videos succeed). As items' information, we use the category of a video so that, e.g., videos belonging to the category *music* correspond to one group  $g$  and videos belonging to *sport* correspond to a different group  $g'$ . Since the membership to a group  $g$  is the only thing that characterizes an item, predictive strategies can only be based on the probability of having  $E$  for that group,  $P(E|g)$ .

In principle, one can think about different strategies on how to issue binary predictions on the items of a group  $g$ . They can be based on the likelihood (L)  $P(E|g)$  or on the posterior (P) probability  $P(g|E)$  [24], and they can issue predictions stochastically (S), with rates proportional to the computed probabilities, or deterministically (D), only for the groups with largest  $P(g|E)$  or  $P(E|g)$ . These simple considerations lead to four (out of many) alternative strategies to predict events (raise alarms) for items in group  $g$

**(LS)** stochastically based on the likelihood, i.e. with probability  $\min\{1, \beta P(E|g)\}$ , with  $\beta \geq 0$ ;

**(LD)** deterministically based on the likelihood, i.e. always if  $P(E|g) > p_*$ , with  $0 \leq p_* \leq 1$ ;

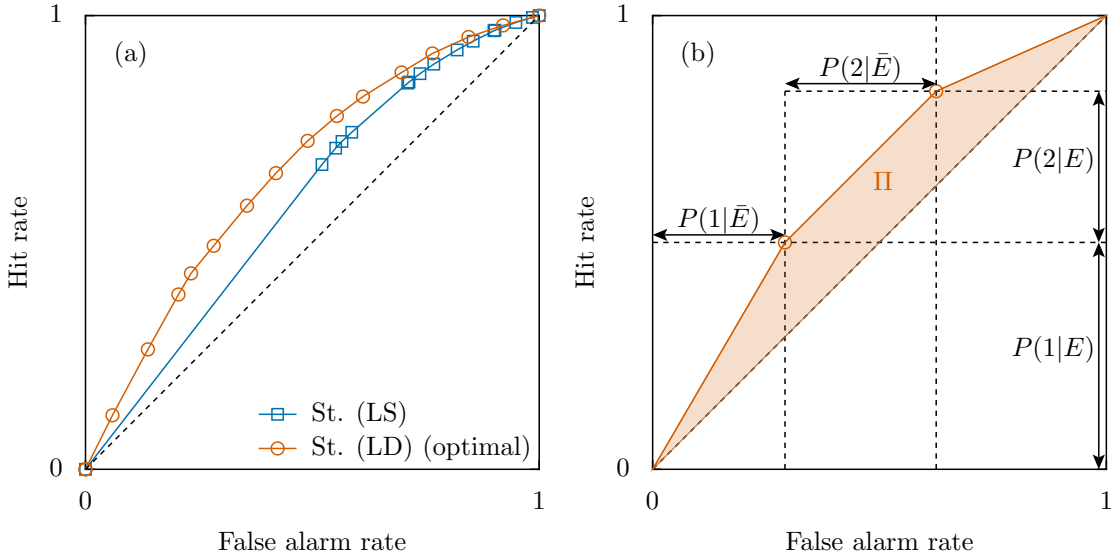
**(PS)** stochastically based on the posterior, i.e. with probability  $\min\{1, \beta' P(g|E)\}$ , with  $\beta' \geq 0$ ;

**(PD)** deterministically based on the posterior, i.e. always if  $P(g|E) > p'_*$ , with  $0 \leq p'_* \leq 1$ .

In the limit of large number of predictions (items), the fraction of events that strategy (LS) predicts for each group  $g$  matches the probability of events  $P(E|g)$  and therefore strategy (LS) is *reliable* [33] and can be considered a natural extension of a probabilistic predictor. Predictions of strategies (LD), (PS) and (PD) do not follow  $P(E|g)$  and therefore they are not reliable.

The quality of a strategy for event prediction is assessed by computing the false alarm rate (or False Positive Rate, equal to one minus the specificity) and the hit rate (True Positive Rate, equal to the sensitivity) over all predictions (items), see Appendix for details. Varying the amount of desired false alarms of the prediction strategy ( $\beta, p_*, \beta'$ , and  $p'_*$  in the examples above), a curve in the hit  $\times$  false-alarm space is obtained, see Fig. 2(a). The overall quality is measured by the area below this curve, known as Area Under the Curve (AUC) [34]. For convenience, we use the area between the curve and the diagonal (hits=false-alarms),  $\Pi = 2\text{AUC} - 1$  (equivalent to the Gini coefficient). In this way,  $\Pi_S \in (-1, 1)$  represents the improvement of strategy  $S$  against a random prediction. In absence of information  $\Pi_S = 0$  and perfect predictions lead to  $\Pi = 1$ . In the YouTube example considered above, we obtain  $\Pi_{PS} < \Pi_{LS} < \Pi_{PD} < \Pi_{LD}$  (17%, 18%, 29%, 32%), indicating that strategy (LD) is the best one.

We now argue that strategy (LD) is optimal (or *dominant* [35]), i.e., for any false alarm rate it leads to a larger hit rate than any other strategy based on the same set of  $P(E|g)$ . To see this, notice that strategy (LD) leads to a piecewise linear curve, see Fig. 2(b), and is the only ordering of the groups that enforces convexity in the hit  $\times$  false-alarms rates space, see Appendix 1.2 for a formal derivation. The ranking of the groups by  $P(E|g)$  implies a ranking of the items, an implicit assumption in the measure of the performance of classification rules [34, 36]. The existence of an optimal strategy implies that the freedom in choosing the prediction strategy argued above is not genuine and that we can ignore the alternative strategies. In our context, it implies that the performance of the optimal strategy measures a



**Figure 2. Quantifying the quality of event-prediction strategies requires measuring both the hit and false alarm rates.** (a) Performance of Strategy (LS) and Strategy (LD) for the problem of predicting views of YouTube videos 20 days after publication based on their categories. The symbols indicate where the rate of issued predictions for a given group equals 1 (the straight lines between the symbols are obtained by issuing predictions randomly with a growing rate). (b) Illustration of the prediction curve (red line) for an optimal strategy with three groups  $g = 1, 2, 3$  with  $P(1) = P(2) = P(3) = 1/3$  and  $P(E|1) = 0.3, P(E|2) = 0.2, P(E|3) = 0.1$ .

property of the system (or problem), and not simply the efficiency of a particular strategy. Therefore, we use the quality of prediction of the optimal strategy ( $\Pi \equiv \Pi_{LD}$ ) to quantify the predictability (i.e., the potential prediction) of the system for the given problem and information. By geometrical arguments we obtain from Fig. 2 (b) (see Appendix)

$$\Pi = \sum_g \sum_{h < g} \frac{P(g)P(h)(P(E|h) - P(E|g))}{P(E)(1 - P(E))}, \quad (2)$$

where  $P(g)$  is the probability of group  $g$  and  $g$  is ordered by decreasing  $P(E|g)$ , i.e.,  $h < g \Rightarrow P(E|h) > P(E|g)$ .

The value of  $\Pi$  can be interpreted as the probability of a correct classification of a pair of  $E$  and  $\bar{E}$  items [34, 36]. In practice, the optimality of this strategy is dependent on the estimation of the ordering of the groups according to  $P(E|g)$ . Wrong ordering may occur due to finite sampling on the training dataset or non-stationarities in the data. In fact, any permutation of indexes in Eq. (2) reduces  $\Pi$ .

## Results

### Application to Data

Here we apply our methodology to the four social-media data described above. We consider the problem of predicting at time  $t_1 \geq 0$  whether the attention  $x$  of an item at time  $t_* > t_1$  will pass a threshold  $x_*$ . In practice, the calculation of  $\Pi$  from the data is done counting the number of items: (i) in each group  $g$

$[P(g) = (\# \text{ items in } g)/(\# \text{ items})]$ ; (ii) that lead to an event  $[P(E) = (\# \text{ items that crossed the threshold } x_* \text{ at } t_*)/(\# \text{ items})]$ ; and (iii) that lead to an event given that they are in group  $g$   $[P(E|g) = (\# \text{ items in } g \text{ that crossed the threshold } x_* \text{ at } t_*)/(\# \text{ items in } g)]$ . Finally, the groups are numbered as  $g = 1, 2, \dots, G$  by decreasing  $P(E|g)$  and the sum over all groups is computed as indicated in Eq. (2). In Ref. [29] we provide a python script which performs this calculation in the data.

We report the values of  $\Pi$  obtained from Eq. (2) considering two different informations on the items:

- 1) the attention at prediction time  $x(t_1)$ ;
- 2) information available at publication time  $t = 0$  (metadata).

In case 1), a group  $g$  corresponds to items with the same  $x(t_1)$ . These groups are naturally ordered in terms of  $P(E|g)$  by the value of  $x(t_1)$  and therefore the optimal strategy is equivalent to issue positive prediction to the items with  $x(t_1)$  above a certain threshold. In case 2), the groups correspond to items having the same meta-data (e.g., belonging to the same category). In this case, we order the groups according to the empirically observed  $P(E|g)$  (as discussed above). Before performing a systematic exploration of parameters, we illustrate our approach in two examples :

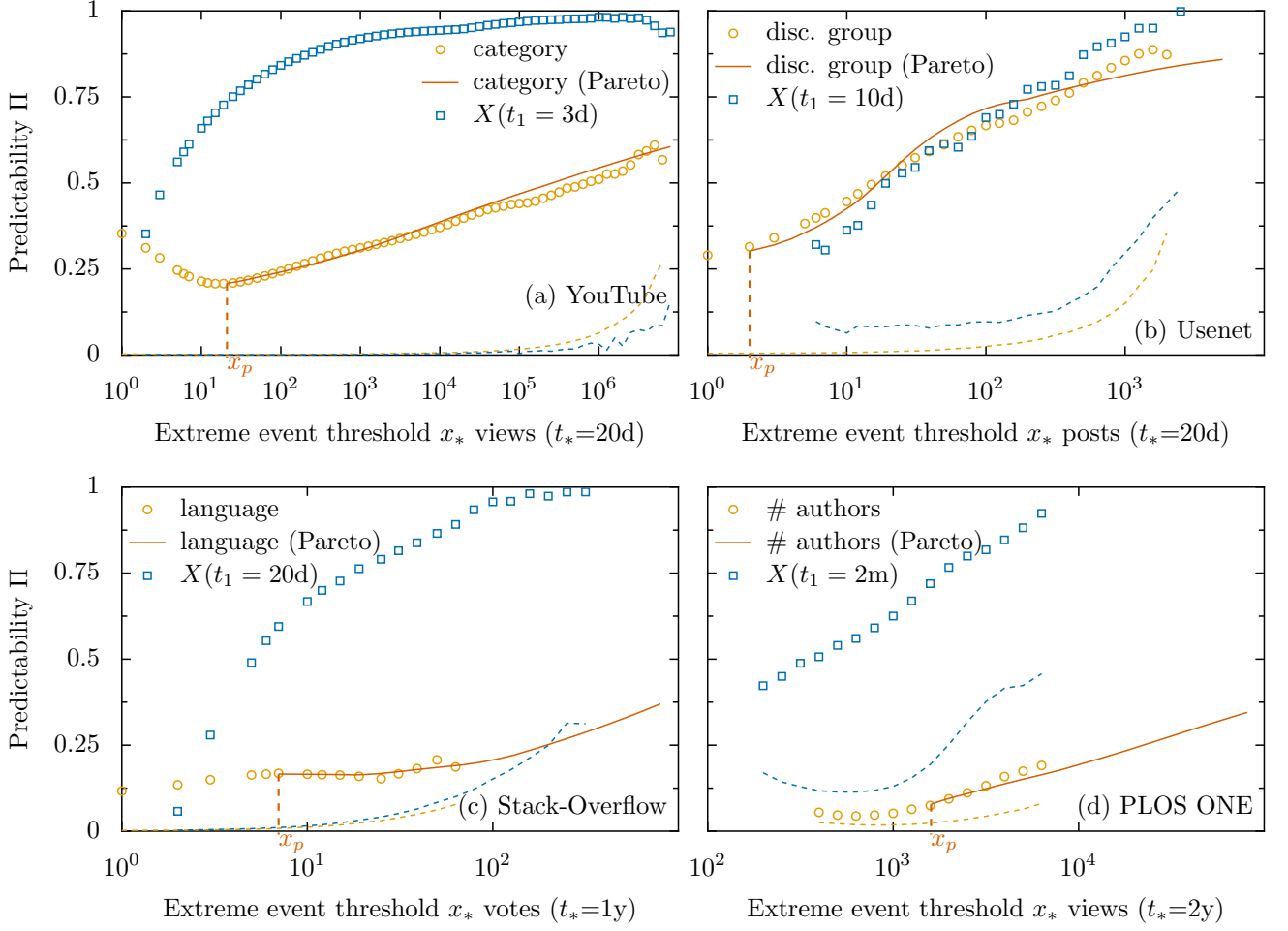
- Consider the case of predicting whether YouTube videos at  $t_* = 20$  days will have more than  $x_* = 1,000$  views. For case 1), we use the views achieved by the items after  $t_1 = 3$  days and obtain a predictability of  $\Pi = 90\%$ . For case 2), we obtain that using the day of the week to group the items leads to  $\Pi = 3\%$  against  $\Pi = 31\%$  obtained using the categories of the videos. This observation, which is robust against variations of  $x_*$  and  $t_*$ , shows that the category but not the day of the week is a relevant information in determining the occurrence of extreme events in YouTube.
- Consider the problem of identifying in advance the papers published in the online journal PLOS ONE that received at least 7500 views 2 years after publication, i.e  $X(t_* = 2\text{years}) > x_* = 7500$  (only  $P(E) = 1\%$  achieve this threshold). For case 1), knowing the number of views at  $t_1 = 2$  months after publication leads to a predictability of  $\Pi = 93\%$ . For case 2), a predictability  $\Pi = 19\%$  is achieved alone by knowing the number of authors of the paper –surprisingly, the chance of achieving a large number of views decays monotonously with number of author ( $g$  increases with number of authors).

The examples above show that formula (2) allows for a quantification of the importance of different factors (e.g., number of authors, early views to the paper) to the occurrence of extreme events, beyond correlation and regression methods (see also Ref. [18]). Besides the quantification of the predictability of specific problems, by systematically varying  $t_1, t_*$ , and  $x_*$  we can quantify how the predictability changes with time and with event magnitude. Our most significant finding is that in all tested databases and grouping strategies the predictability increases with  $x_*$ , i.e., extreme events become increasingly more predictable, as shown in Fig. 3.

## Discussion

We now explain why predictability increases for extreme events (increasing  $x_*$ ). We first show that this is not due to the reduction of the number of events  $P(E)$ . Consider the case in which  $E$  is defined in the interval  $[x_f - \Delta x, x_f + \Delta x)$ . Assuming  $P(X)$  to be smooth in  $X$ , for  $\Delta x \rightarrow 0$  at fixed  $x_f$  we have that  $P(E) \rightarrow P(x_f)\Delta x$  and  $P(E|g) \rightarrow P(x_f|g)\Delta x$  ( $P(g)$  remains unaffected), and Eq. (2) yields

$$\Pi = \frac{\sum_g \sum_{h>g} P(g)P(h) (P(x_f|h) - P(x_f|g))}{P(E_f)[1 - \Delta x P(x_f)]}, \quad (3)$$



**Figure 3. Predictability increases for extreme events.** If the attention an item receives at time  $t_*$  is above a threshold,  $X(t_*) > x_*$ , an event  $E$  is triggered. The plots show how the predictability  $\Pi$  changes with  $x_*$  using two different informations to combine the items in groups  $\{g\}$ . **Black circles:**  $\Pi$  at time  $t = 0$  using metadata of the items to group them. The **red lines** are computed using as probabilities  $P(E|g)$  the Extreme Value distribution fits for each group at a threshold value  $x_p$ , see Eq. (1) and SI Sec. 2. **Blue squares:**  $\Pi$  at time  $t_1 < t_*$  using  $X(t_1)$ , i.e., the attention the item obtained at day  $t_1$ . The **dashed lines** are the values of the 95% percentile of the distribution generated by measuring  $\Pi$  in an ensemble of databases obtained shuffling the attribution of groups ( $g$ ) to items (the colors match the symbols and symbols are shown only where  $\Pi$  is at least twice this value). Results for the four databases are shown: **(a)** YouTube ( $X$ : views of a video; metadata: video category); **(b)** Usenet discussion groups ( $X$ : posts in a thread; metadata: discussion group of the thread); **(c)** Stack-Overflow ( $X$ : votes to a question; metadata: programming language of the question, see SI Sec. 2 for details); **(d)** PLOS ONE ( $X$ : online views of a paper; metadata: number of authors of the paper).

which decreases with  $\Delta x \rightarrow 0$ . This shows that the increased predictability with  $x_*$  is not a trivial consequence of the reduction of  $P(E)$  ( $\Delta x \rightarrow 0$ ), but instead is a consequence of the change in  $P(E|g)$  for extreme events  $E$ .

Systematic differences in the tails of  $P(X|g)$  lead to an increased predictability of extreme events. Consider the case of two groups with cumulative distributions  $P(E|g)$  that decay as a power law as in Eq. (1) with exponents  $\alpha$  and  $\alpha' = \alpha + \epsilon$ , with  $P(1) = P(2)$ . From Eq. (2),  $\Pi$  for large  $x_*$  ( $1 - P(E) \approx 1$ ) can be estimated as

$$\Pi = \frac{1}{4} \frac{P(E|1) - P(E|2)}{P(E|1) + P(E|2)} = \frac{1}{4} \frac{x_*^{-\alpha} - x_*^{-(\alpha+\epsilon)}}{x_*^{-\alpha} + x_*^{-(\alpha+\epsilon)}} \approx \frac{1}{8} \log(x_*)\epsilon, \quad (4)$$

where the approximation corresponds to the first order Taylor expansion around  $\epsilon = 0$ . The calculation above can be directly applied to the results we obtained issuing predictions based on metadata. The logarithmic dependency in Eq. (4) is consistent with the roughly linear behavior observed in Fig. 3(a,b). A more accurate estimation is obtained using the power-law fits of Eq. (1) for each group  $g$  and introducing the  $P(E|g)$  obtained from these fits in Eq. (2). The red line in Fig. 3 shows that this estimation agrees with the observations for values  $x_* \gtrsim x_p$ , the threshold used in the fit. Deviations observed for  $x_* \gg x_p$  (e.g., for PLOS ONE data in panel (d)) reflect the deviations of  $P(E|g)$  from the Pareto distribution obtained for small thresholds  $x_p \ll x_*$ . This allows for an estimation of the predictability for large thresholds  $x_*$  even in small datasets (when the sampling of  $E$  is low).

A similar behavior is expected when prediction is performed based on the attention obtained at short times  $t_1$ . Eq. (3) applies in this case too and therefore the increase in predictability is also due to change in  $P(E|g)$  with  $x_*$  for different  $g$  (and not, e.g., due to the decrease of  $P(E)$ ). For increasingly large  $x_*$  the items with significant probability of passing threshold concentrate on the large  $x(t_1)$  and increase the predictability of the system. We have verified that this happens already for simple multiplicative stochastic processes, such as the geometric Brownian motion (see Fig. S2). This provides further support for the generality of our finding. The dynamics of attention in specific systems affect the shape of predictability growth with threshold.

Altogether, we conclude that the difference in (the tails of) the distribution of attention of different groups  $g$  is responsible for the increase in predictability for extreme events: for large  $x_*$ , any informative property on the items increases the relative difference among the  $P(E|g)$ . This corresponds to an increase of the information contained in the grouping which leads to an increase in  $\Pi$ .

## Conclusions

In summary, we propose a method, Eq. (2), to measure the predictability of extreme events for any given available information on the items. We applied this measure to four different social media databases and quantified how predictable the attention devoted to different items is and how informative are different properties of the items. We quantified the predictability due to metadata available at publication date and due to the early success of the items and found that usually the latter quickly becomes more relevant than the former<sup>1</sup>. Our most striking finding is that extreme events are better predictable than non-extreme events, a result previously observed in physical systems [25] and in time-series models [24, 28]. For social media, this finding means that for the large attention catchers the surprise is reduced and the possibilities to discriminate success enhanced.

These results are particularly important in view of the widespread observation of fat-tailed distributions of attention, which imply that extreme events carry a significant portion of the total public attention. Similar distributions appear in financial markets, in which case our methodology can quantify the increase in predictability due to the availability of specific information (e.g., in Ref. [37] Internet activities were used as information to issue predictions). For the numerous models of collective behavior

<sup>1</sup>Our results can also be applied for combinations of different informations on the items (e.g., a group  $g$  can be composed by videos in the category *music* with a fixed  $x(t_1)$ ). In practice, the number of groups  $G$  should be much smaller than the observations in the training dataset to ensure an accurate estimation of  $P(E|g)$ .



leading to fat tails [6, 8–11, 17, 18], the predictability we estimate is a bound to the quality of binary event predictions. Furthermore, our identifications of the factors leading to an improved predictability indicate which properties should be included in the models and which ones can be safely ignored (feature selection). For instance, the relevant factors identified in our analysis should affect the growth rate of items in rich-get-richer models [11, 12] or the transmission rates between agents in information-spreading models [38]. The use of  $\Pi$  to identify relevant factors goes beyond simple correlation tests and can be considered as a measure of causality in the sense of Granger [39].

Predictability in systems showing fat tails has been a matter of intense debate. While simple models of self-organized criticality suggest that prediction of individual events is impossible [5], the existence of predictable mechanisms for the very extreme events has been advocated in different systems [26]. In practice, predictability is not an yes/no question [7, 22] and the main contribution of this paper is to provide a robust quantification of the predictability of extreme events in systems showing fat-tailed distributions.

## 1 Appendix

### 1.1 Quality of binary predictions

Comparing binary predictions and observations gives four possible results, given by the combination of the prediction (positive or negative) and its success (true or false). If  $A$  denotes the prediction of an event (an alarm), the hit rate (or True Positive Rate) and the false alarm rate (or False Positive Rate) are defined as

$$\begin{aligned} \text{hit rate} &\equiv \frac{\text{number of true positives}}{\text{number of positives}} = P(A|E), \\ \text{false alarm rate} &\equiv \frac{\text{number of false positives}}{\text{number of negatives}} = P(A|\bar{E}). \end{aligned} \tag{5}$$

These are analogous to measures like Accuracy and Specificity or Precision and Recall. Prediction strategies typically have a specificity parameter (e.g., controlling the rate of false positives). Varying this parameter, a prediction curve that goes from  $(0, 0)$  to  $(1, 1)$  is built in the hit $\times$ false-alarm space.

### 1.2 Demonstration that strategy LD (Bayes classifier) is dominant

A strategy is dominant when for any given false alarm rate, the hit rate is maximized. Following definition (5), we write the  $x$  and  $y$  coordinates of the hit $\times$ false-alarm plot as

$$\begin{aligned} \text{hit rate} &\equiv P(A|E) = \sum_{g=1}^G P(A|g)P(g|E) = \sum_{g=1}^G \pi_g y_g \equiv y, \\ \text{false-alarm rate} &\equiv P(A|\bar{E}) = \sum_{g=1}^G P(A|g)P(g|\bar{E}) = \sum_{g=1}^G \pi_g x_g \equiv x, \end{aligned} \tag{6}$$

where for notational convenience  $y_g \equiv P(g|E)$ ,  $x_g \equiv P(g|\bar{E})$ , and  $\pi_g \equiv P(A|g)$ . Since predictions are issued based only on the information about the groups, strategies (both deterministic and stochastic) are defined uniquely by  $\pi_g$ , while  $x_g$  and  $y_g$  are estimated from data. The computation of the dominant strategy corresponds to finding the  $\pi_g$ 's that maximize  $y$  with the constraint  $\sum_{g=1}^G \pi_g x_g = x$ . This problem can be solved exactly by applying the simplex method. Define  $h$  such that  $\sum_{g < h} x_g < x < \sum_{g \leq h} x_g$ .

$\sum_{g \leq h} x_g$ ; we write Eq. (6) as:

$$\begin{aligned} y - \sum_{g < h} y_g &= - \sum_{g < h} (1 - \pi_g) y_g + \sum_{g > h} \pi_g y_g + \pi_h y_h, \\ x - \sum_{g < h} x_g &= - \sum_{g < h} (1 - \pi_g) x_g + \sum_{g > h} \pi_g x_g + \pi_h x_h. \end{aligned} \quad (7)$$

Isolating  $\pi_h$  in the lower equation and introducing it in the top one we obtain

$$y = \sum_{g < h} y_g + x \frac{y_h}{x_h} \quad (8)$$

$$- \sum_{g < h} (1 - \pi_g) x_g \left( \frac{y_g}{x_g} - \frac{y_h}{x_h} \right) + \sum_{g > h} \pi_g x_g \left( \frac{y_g}{x_g} - \frac{y_h}{x_h} \right). \quad (9)$$

Notice that  $y_g/x_g$  is the contribution of the group  $g$  to the slope of the prediction curve in the hit  $\times$  false-alarm space. If the  $G$  groups are ordered by decreasing  $P(E|g)$ , then  $y_g/x_g$  also decreases with  $g$ . Therefore  $(y_g/x_g - y_h/x_h) > 0$  for  $g < h$  and  $(y_g/x_g - y_h/x_h) < 0$  for  $g > h$  and Eq. (8) is maximized by choosing  $\pi_g$  such that the two last terms vanish. This is achieved choosing

$$\pi_g = \begin{cases} 1 & g < h, \\ \frac{x - \sum_{g < h} x_g}{x_h} & g = h, \\ 0 & g > h, \end{cases} \quad (10)$$

which correspond to issuing positive predictions only to the  $h$  groups with largest <sup>2</sup>  $P(E|g)$  and is equivalent to strategy (LD) mentioned in the main text.

### 1.3 Computation of $\Pi$ for the optimal strategy

As illustrated in Fig. 2(b), the partition performed by the optimal strategy defines  $G$  different intervals in the hit and false alarm axis (the points for which  $P(E|g) = P_*$ ,  $g \in \{1 \dots G\}$ ) and therefore  $G^2$  rectangles in the hit  $\times$  false-alarm space. The  $(g, h)$  rectangle has height  $P(h)P(E|h)/P(E) = P(h|E)$ , width  $P(g|\bar{E})$  (where  $\bar{E}$  is the complement of  $E$ , i.e.,  $P(\bar{E}|g) = 1 - P(E|g)$ ), and therefore it has an area  $A_{g,h} = P(h|E)P(g|\bar{E})$ . The curve of strategy (LD) is the union of the diagonals of the  $g = h$  rectangles (which are obtained by increasing  $p_*$ ).  $\Pi$  is two times the sum of the rectangles and triangles under this curve minus half of all the area:

$$\begin{aligned} \Pi &= 2 \left[ \sum_g \sum_{h < g} A_{g,h} + \frac{1}{2} \sum_g A_{g,g} - \frac{1}{2} \sum_g \sum_h A_{g,h} \right] \\ &= \sum_g \sum_{h < g} A_{g,h} - \sum_g \sum_{h > g} A_{g,h} \\ &= \sum_g \sum_{h < g} (A_{g,h} - A_{h,g}) \\ &= \sum_g \sum_{h < g} P(h|E)P(g|\bar{E}) - P(h|\bar{E})P(g|E) \\ &= \frac{\sum_g \sum_{h < g} P(g)P(h) (P(E|h) - P(E|g))}{P(E)(1 - P(E))}, \end{aligned} \quad (11)$$

where we used  $\sum_g \sum_h A_{g,h} = 1$ . This finishes our demonstration of Eq. (2).

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<sup>2</sup>Positive events are predicted for the group  $h$  in Eq. (10) as much as needed to reach the required false positive rate  $x$ .

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